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FLAT SUPERSONIC UNDEREXPANDED JETS USING A LASER SCHLIEREN METHOD

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1. Interest in flat jet flows arose due to progress in nonequilibrium physicochemical kinetics, creation of gasdynamic lasers (GDL), and solution of other problems arising in the new technology. In particular, in modeling GDL, flat jets have certain advantages over the usual nozzles: for example, they provide maximum expansion velocity of the flow with planar geometry. However, the interrelation of the kinetic and gasdynamic processes occurring in the jets, as well as the presence of viscous effects, manifested, for example, in the formation of boundary layers, complicate the study of supersonic, high-enthalpy, gas jets and require that these jets be experimentally studied.

The purpose of this work is to investigate gasdynamic characteristics of a flat jet: experimental determination of the density profile along the center of the stream tube of the jet and numerical estimates of the boundary layer, arising on the lateral surfaces bounding the jet, based on a theoretical analysis. The proposed experimental method, which has high sensitivity and temporal resolution $\leq 1\,\mu$sec, is based on measuring a sequence of density gradients, relating to different cross sections of the flow studied with the help of the laser schlieren method [1].

There are several papers concerned with investigation of flat jets flowing out of a sonic slit nozzle into a space bounded by two parallel surfaces [2-5]. The wave structure of such a stationary flow was studied by the shadow method in [2-4]. A generalizing dependence of the location of the central jump as a function of the determining parameters was obtained in [4]. The results are compared with data for axisymmetrical jets. In some regimes, separation of the boundary layer, forming on the lateral surfaces bounding the jet, is observed [2]. The flow field of a flat, weakly underexpanded perfect gas jet is calculated in [5] using the stabilization method.

From an analysis of the theoretical model both for flat and for axisymmetrical jets, it is possible to obtain a generalized relation for the density distribution at the center of the stream tube [3]:

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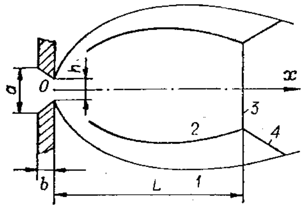

Fig. 1

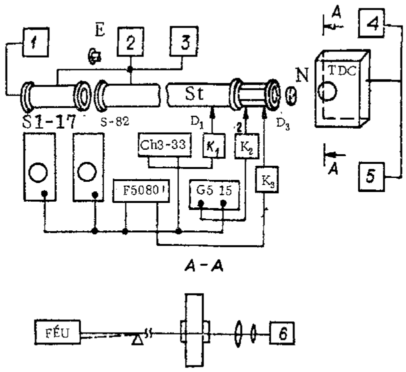

Fig. 2

$$\rho(x)/\rho_e = B(x/d_e)^{-(j+1)}, \qquad (1.1)$$

where x is the distance along the Ox axis from the nozzle cutoff, and the origin of the axis is situated at the center of the critical section (Fig. 1); $\rho(x)$ is the density at distance x; d_e is the diameter of the nozzle cutoff in the case of an axisymmetrical jet or slit height h for a flat jet; j = 1 and 0 for flat and axisymmetrical jets, respectively; B is a coefficient that depends on the composition and thermodynamic state of the gas, as well as the flow geometry; e is an index relating to parameters at the nozzle cutoff.

In Fig. 1, the boundary of the jet is indicated by the number 1, while the numbers 2-4 correspond to the suspended, closing, and reflected shocks.

The results of the experimental investigation of a free jet, flowing out of a slit nozzle, indicate the two-dimensional nature of the flow [6], but the disagreement between the measured values of the density and the corresponding calculations attains ~80-100%. In [6], an electron-beam technique was used to determine the density and the calculations were performed by the method of characteristics.

In [7], an electron-beam probe was used to establish the validity of the assumptions as to the type of flow from the source in the near-axial stream tube of an axisymmetrical jet for coordinate x greater than several units, i.e., the validity of relation (1.1) for the regimes examined.

One of the problems addressed in this investigation is to determine the possibility of using a model flow from a cylindrical source to describe the flow in a flat jet.

2. The experiments were performed in a two-dimensional chamber (TDC), which was connected to the end-face of a shock tube (ST) (Fig. 2). The inner diameter of the tube was 50 mm, the length of the low-pressure chamber (LPC) was 3 m, and the length of the high-pressure chamber (HPC) was 1.5 m. The closing section of the LPC was the observational section of length 200 mm.

Helium was used as the pushing gas. The velocity of the shock wave was measured at two base lines 266 and 94 mm using piezoelectric sensors S_1, S_2, and S_3 (Fig. 2) to within 2%. Within the limits of the error indicated, the velocity of the incident wave was constant on the baselines S_1-S_3 and in addition the distance between the end-face and the sensor S_3 was 40 mm. The following notation is used in Fig. 2: 1) system for injecting gas into the HPC; 2) the system for evacuating the ST; 3) system for injecting gas into LPC; 4) system for injecting gas into the TDC; 5) system for evacuating the TDC; 6) OKG-13 laser, E indicates knife edges cutting the diaphragm; K_1, K_2, and K_3 are the emitter repeaters; G5-15 is a pulse generator; F5080 and Ch3-33 are frequency meters; and, S8-2 and S1-17 are oscillographs.

The two-dimensional chamber (TDC) with the slit sonic nozzle (N) was intended to create and study pulsed two-dimensional jets and had the shape of a rectangular parallelepiped with the following inner dimensions: height, 480 mm; length, 280 mm; width, 45 mm. The plane of symmetry, in which the axis of the tube and the center line of the critical section of the nozzle are located, is perpendicular to the two parallel side walls bounding the jet and separate the TDC in half along its height. The nozzle gap of width h = 1.0 mm or h = 0.3 mm was cut in the nozzle insert (N). The input slit width and length of the contracting part were a = 5 mm and b = 3.4 mm (Fig. 1), respectively. Windows with diameter 75 mm, making it possible to probe the jet optically, were placed symmetrically directly behind the nozzle in the sidewalls of the TDC.

To measure the density gradients, the well-known highly sensitive quantitative laser schlieren method was used. This method has recently been widely used in investigations using shock tubes and also for studying more complicated flows, for example, boundary layers in a hypersonic flow [8]. The theory of the method is described quite completely in [1, 9].

An OKG-13 helium–neon laser was sued as the source of the laser beam in our investigations (Fig. 2) (λ = 632.8 nm). The diameter of the laser beam (distance between e^{-2} points) was decreased from ~1.5 to ~0.8 mm with the help of a simple telescopic system consisting of two lenses situated between the laser and the flow being studied. The divergence of the beam was $4 \cdot 10^{-4}$ rad.

The laser beam passed in the plane of symmetry perpendicular to the Ox axis through the window of the TDC and was incident at a distance 4 m on a Foucault knife edge, placed in front of the FÉU-51 radiation detector. Light filters, limiting the transmission band, a neutral filter, decreasing the intensity of the laser radiation, as well as a matted diffusely scattering screen, to illuminate the photocathode more uniformly, were placed directly in front of the FÉU. The entire optical part (lenses, mirrors, knife edge, as well as the laser and FÉU), to avoid transmission of vibrations from the shock tube and pumps, was placed on a special bracket not connected to the shock tube and fixed on the wall.

The sensitivity of the experiments permitted recording density gradients ~10^{-6} g/cm^4. The estimated linear displacement of the beam in the flow in the entire range of gradients recorded did not exceed 0.02 mm.

The experiments were performed with carbon dioxide gas (CO_2) and a mixture of carbon dioxide gas with nitrogen ($0.3CO_2 + 0.7N_2$), where 0.3-07 are the molar fractions of CO_2 and N_2 in the mixture, respectively. The thermodynamic parameters in the shock-heated gas plug behind the reflected shock wave at the end-face of the shock tube were calculated from the measured velocities of the incident and reflected shock waves and are presented in Table 1. The velocities of the reflected shock wave, measured on the base between the end-face and the sensor S_3, were less than the values computed using the ideal theory. The difference constituted 22–30% depending on the regime. Equilibrium temperatures, obtained with the help of the measured velocity of the shock wave, were 4–6% lower than those computed from the velocity of the incident wave and are presented in Table 1.

Table 1 also contains information on the initial pressure in the two-dimensional chamber p_∞ and the coordinate sections $\bar{x} = x/h$ at the center of the stream tube of the jet, in which the measurements were performed. For each regime, indicated in Table 1, a series of 3–5 experiments were performed in a single section.

To eliminate possible systematic errors, the measurements were performed using different models of lasers, FÉU, emitter repeaters, and oscillographs, one of which had memory, and the magnitude of the signal recorded was obtained by averaging two oscillograms.

The relative error of a separate measurement of the density gradient $d\rho(x)/dx$ was not more than 12.5% and was determined primarily by noise in the signal.

The quasistationary stage of efflux of the jet, when in the measured sections a stationary value of the signal recorded was established, was studied. The time for establishing the quasistationary efflux regime was ~150–200 μsec.

Analysis of the experimental data using statistical methods showed that for each regime (see Table 1) $\lg|d\rho/dx|$ as a function of $\lg x$ can be represented in the interval studied by a linear regression with significance coefficient 0.95 and, thus, the dependences of $d\rho/dx$ on x are expressed by a power-law function.

After the value of the gradient $d\rho/dx$ obtained in each separate experiment was put into dimensionless form with the help of the density ρ_0 and the height of the slit h, the experimental points, related to different regimes, were grouped so that it was possible to describe the profile of variations in the gradient $d\bar{\rho}/d\bar{x}$ of a single function of $\bar{x}$ for each gas and slit size h:

$$d\bar{\rho}/d\bar{x} = B(\bar{x})^{-\beta}, \tag{2.1}$$

where $\bar{\rho} = \rho(x)/\rho_0$.

The 95% confidence interval found permitted determining the interval of values of $d\bar{\rho}/d\bar{x}$, which, for example, for a CO_2 jet from a slit h = 1.0 mm with $\bar{x}$ = 10; and 15 constituted 8 and 10% of the value of the function, respectively.

The density profile $\bar{\rho}(\bar{x})$ was determined by integrating relation (2.1):

$$\bar{\rho}(\bar{x}) = C_1(\bar{x})^{-\gamma} + C_2, \tag{2.2}$$

TABLE 1

Slit width h, mm	1,0				0,3
Composition of outflowing gas	CO_2		$0{,}3CO_2+0{,}7N_2$		CO_2
Velocity of incident shock wave-km/sec	1,32	1,03	1,45	1,34	1,32
T_0, K	1800	1250	2070	1810	1800
ρ_0, g/cm³	$3{,}88\cdot10^{-3}$	$2{,}67\cdot10^{-3}$	$1{,}67\cdot10^{-3}$	$1{,}51\cdot10^{-3}$	$3{,}88\cdot10^{-3}$
p_0, Pa	$1{,}26\cdot10^{6}$	$6{,}1\cdot10^{5}$	$8{,}2\cdot10^{5}$	$6{,}7\cdot10^{5}$	$1{,}26\cdot10^{6}$
p_∞, Pa	$5\cdot10^{3}$		$5\cdot10^{3}$		$5\cdot10^{3}$
$\bar{x}$	5; 10; 15		5; 10; 15		17; 33; 40

where $C_1 = B/(1-\beta)$; $\gamma = \beta - 1$; C_2 is a constant of integration. The density distribution at the center of the stream tube is known to within the constant C_2, whose value must be determined from boundary or other physical conditions.

To solve the problem of determining the constant C_2, we shall examine the density variation in the central stream tube for flows close to the one examined. As shown in [10], the presence of nonequilibrium physicochemical processes, occurring in jets, has little effect on the density profile. For this reason, a simple model, which, possibly, will permit estimating the distribution $\bar{\rho}(\bar{x})$ in the central stream tube, is represented as a model of cylindrical expansion of a perfect gas. From the calculation it follows that for any $\gamma = c_p/c_v$ the profile of $\bar{\rho}(\bar{x})$ is approximated by a linear function in logarithmic coordinates, beginning with $\bar{x} = 4$ (see, for example, curve 4 in Fig. 3), i.e., the density distribution can be expressed as a power-law function in the form

$$\bar{\rho}(\bar{x}) = A(\bar{x})^{-\alpha}. \tag{2.3}$$

In addition, it follows from an analysis of a series of experimental and computational papers [5-7, 11, 12], in which jets and flows were studied in two-dimensional nozzles with a large flare angle, that for continuum flow in the central tube of the flow and for $\bar{x} \geq 2$-4, the density profile along the stream tube indicated can be approximated by a power-law function of the form (2.3). The corresponding values of A and α are presented in Table 2.

Thus, starting from an analysis of the indicated papers, it was assumed that in the jet studied, in the region near the axis for coordinates $\bar{x} \geq 5$, the change in density is also approximated by a power law function (2.3) and for this reason the integration constant C_2 in (2.2) can be set equal to zero.

3. The profiles of the relative density distribution obtained $\bar{\rho}(\bar{x})$ are represented in logarithmic coordinates in Fig. 3. The functions 1 and 2 correspond to jets of CO_2 and the mixture ($0.3CO_2 + 0.7N_2$) for a slit h = 1.0 mm, while the function 3 was obtained for the CO_2 jet and h = 0.3 mm. The values of the coefficients A and α in the approximating dependences of the type (2.3), as well as the corresponding gas dynamic efflux conditions are presented in Table 2. This table also presents analogous data taken from papers in which the flow was studied in flat jets [5], in free jets flowing out of a slit nozzle [6], in two-dimensional nozzles [11, 13], as well as in axisymmetrical jet flowing out of a supersonic nozzle [7]. Table 2 presents the distributions obtained by the method of natural coordinates [12] and the results of the calculation of a model of cylindrical expansion.

The ratio of the sides of the slit is indicated by l/h; the jet height-to-width ratio is indicated by L/h (see Fig. 1). For an axisymmetrical jet, the characteristic size is the diameter of the output section of the supersonic nozzle d_e.

It follows from a comparison of the data in Table 2 that the coefficient characterizing the degree of expansion of the flows varies from 0.6-0.7 for two-dimensional nozzles up to ~ 2 for an axisymmetrical jet. For the distributions in this work, coefficients α = 0.957-1.02 are obtained, which are less than the corresponding

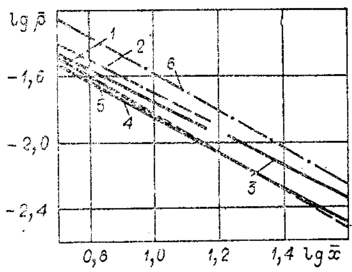

Fig. 3

computed values [12], as well as the values found experimentally in [6] for measurements in a free jet. The fact that the density profile at the center of the stream tube of the nozzle with a large flare angle (70°) has $\alpha > 1$ [11] can be explained by the fact that only the initial section of the flow, which could correspond to a non-steady state stage of the efflux, is examined. In this case, as is well known [13], the variation in density $\bar{\rho}(\bar{x})$ corresponds to a steeper dependence.

It follows from a comparison of the data presented in Fig. 3 that profile 1 is situated below profile 2 by 10–15%. This is evidently related to the deeper expansion of CO_2 on the section of the jet before freezing of the vibrational degrees of freedom. Curves 1 and 3 were obtained for identical efflux regimes of CO_2, but for different h and $\bar{x}$. Profile 3 is situated above the extrapolated profile in region x = 17–40 of curve 1 by 8–10%, which could be related to the different magnitude of the boundary layer, arising on the surfaces bounding the jet and different degree of freezing of the flows.

Distribution 4 was obtained from the cylindrical expansion model for $\gamma = 1.4$, while 5 was obtained by the method of natural coordinates [12], likewise for $\gamma = 1.4$. The curves 4 and 5 intersect approximately at the center of the interval of variation of $\bar{x}$ examined, while at the edges the difference does not exceed 12%. But, both are situated below the experimental curves (1–3). The disagreement, depending on the distributions compared and the coordinates $\bar{x}$, ranges from 6 to 50%. The smallest differences occur for functions 1 and 5 at $\bar{x} = 5$, but already at $\bar{x} = 15$ it constitutes ~20%, while comparison of the dependences 2 and the coinciding dependences 4 and 5 at $\bar{x} = 15$ gives a difference of ~45%.

It likewise follows from an examination of Fig. 3 that the experimental distribution for a free jet 6 lies higher than the distribution of this work (1–3), although in a free jet a deeper expansion should occur. The reason for this disagreement is not entirely known, but it could be a result of condensation of the cold jet (T_0 = 293°K) [6] and the appearance of a systematic error in the measurements, if the effect of the condensate on the electron-beam measurements was not included.

Thus the density distributions found along the central stream tube differ considerably both from the distributions computed using the model [12] and obtained from an analysis of the cylindrical expansion model. The difference attains 50–60% and is evidently due to the influence of real gas properties, primarily viscosity, on the flow.

4. Let us estimate the size of the boundary layer forming in the flows under study. This boundary layer could be the reason for the deviation of the flows from a two-dimensional flow and can even lead in some regimes [2] to separation of the flow from the walls in the flow region near the central shock.

We shall be interested in the boundary layer arising in the plane of symmetry of the jet, when the transverse gradient of thermodynamic parameters can be neglected.

To estimate the size of the boundary layer under conditions of quasistationary flow, the closest model is a model in which an accelerating compressible gas flow with gradient along the flow along a flat semiinfinite plate is examined. The conditions along the edge of the plate correspond to the conditions in the critical section of the nozzle. The boundary layer in the sub- and transonic regions was negelcted.

The nature of the boundary layer was determined by the Reynolds number of the source in the critical section of the nozzle $Re_* = \rho_0 u_* h/\mu_0$, where ρ_0 and μ_0 are the density and viscosity of the gas under stagnation conditions.

Under the conditions of the given experiments $Re_* \simeq 10^4$, which is typical for gasdynamic lasers and less than the Reynolds number of the flow transforming into turbulent flow ($Re_p \simeq 10^6$–10^7 [14, 15]). Thus, a laminar boundary layer is examined.

TABLE 2

Type of flow	Flow of outflowing gas	h, mm	l/h	L/h	Interval of variation of $\bar{x}$	A	α	References
Flat jet	CO_2	1,0	45	170	5—15	0,180	1,02	Expt. of present work
	$0,3CO_2+0,7N_2$	1,0	45	93	5—15	0,179	0,957	
	CO_2	0,3	150	170	17—40	0,17	0,97	
Two-dimensional jet	$\gamma=1,4$	—	∞	—	5—10	0,217	1,167	Calc. using natural coordinates [12]
	$\gamma=1,18$	—	∞	—	5—10	0,181	1,216	
Two-dimensional jet	$\gamma=1,4$	—	∞	—	5—20	0,16	1,05	Model of cylindrical expansion
	$\gamma=1,18$	—	∞	—	5—20	0,147	1,09	
Flat jet	N_2	—	∞	5	1,3—3	0,295	1,18	Calc. using stabilization method [5]
Free jet	Air	0,11	200	51	3—50	0,192	1,1—1,2	Calc. using the method of characteristics [6]
	Air	0,11	200	77	3—70	0,27—0,4	1,1	Experiment, [6]
Flow in flat nozzle (α = =70°)	$0,16CO_2+0,83N_2+$ $+0,01H_2$	0,6	250	—	1,9—3,2	0,167	1,12	» [11]
Flow in flat nozzle (α = =30°)	N_2	6	20	—	2—10	0,5	0,634	» [13]
Axisymmetrical jet	N_2	d_a=3,2—10	—	100	7—40	0,017	2,0	» [7]

As follows from an analysis of [16, 17], when Re_* is of the order of 10^5, the effects of viscosity on the characteristics of the GDL (CO_2) can be neglected, while for $Re_* \sim 10^4$ or less, viscosity leads to losses exceeding 10%.

Thus, in studying flow data, it is necessary to take into account the possible effect of the boundary layer on the jet.

As a rule, the system of equations for the boundary layer can be solved only numerically. For estimates, it is useful to use approximate analytic solutions or approximations.

An approximate solution for growth of the laminar boundary layer under conditions close to the model under examination was obtained in [15]. The velocity profile near the axis, necessary for the calculation, was found from the experimentally determined density distribution in the jet assuming an isentropic flow. The calculation showed that the boundary layer grows by an amount ~1.2 mm both for CO_2 and for the mixture 30% CO_2 with 70% N_2 at a distance $\bar{x} = 25$ for efflux from a slit h = 1 mm and ~2.5 mm for CO_2 at a distance $\bar{x} = 60$ for efflux from a 0.3-mm slit.

The boundary layer forming on the lateral surface of the nozzle or the plate bounding the jet was calculated numerically in [18]. A flow model was assumed in which the flow separated into two parts. The first part involves cylindrical expansion of the gas to the final value M_∞, where $M = u/a$ is the Mach number, u is the velocity of the flow, and a is the velocity of sound, and the second part is an ideal two-dimensional flow with M_∞. The growth of the boundary layer was examined only along the line of intersection of the plane of symmetry with the lateral surface in the absence of transverse flows. For typical GDL conditions ($Re_* \approx 3 \cdot 10^4$) and planar geometry for CO_2 lasers [18], approximating dependences were obtained for $M_\infty = 4.7$ ($\bar{x} > 13$)

$$\xi = 5.25 \cdot 10^{-3}(x/h - 13) + 0.0244, \quad \delta^*/h = 1.13(\xi)^{1/2}, \tag{4.1}$$

where δ^* is the thickness of the region of momentum expulsion.

Under the conditions of the jets studied, M_∞, estimated from the maximum possible velocity $u_{max} = \sqrt{2H}$, where H is the enthalpy of the gas in the "plug" at the end-face of the shock tube, constitutes ≈ 5. It is thus possible to estimate the quantity δ^* from relations (4.1) for conditions realized in experiments. The boundary layer growth δ is ≈ 3 times greater than δ^* and is estimated to be ~1.5 mm both for CO_2 and for the mixture (30% CO_2 + 70% N_2) at the distance $\bar{x} = 25$ with efflux from a h = 1-mm slit and ~3 mm for CO_2 at a distance $\bar{x} = 60$ with efflux from a 0.3-mm slit.

Thus, estimates of the boundary layer arising based on the two models give close values. It follows from the estimates that due to the existence of a boundary layer, the width of the jet in the range of x studied can decrease by 3-10%. Such changes must be taken into account in measuring density gradients in the jet studied, as well as in spectroscopic sounding of the jet.

Thus, the flat supersonic jets studied cannot be assumed to be two-dimensional, while the boundary layer that arises must be taken into account in transmission measurements, since it attains appreciable sizes.

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HIGH-SPEED GAS MOTION IN A POROUS MEDIUM

A. P. Ershov

UDC 532.546

The problems of motion of a gas in a porous medium have been solved repeatedly, starting with the work of L. S. Leibenzon, and mainly relating to filtration of gas in beds. Reference [1] has reviewed this topic. As a rule, the Darcy resistance law is used, valid for small flow velocities. Even in this formulation the gas compressibility leads to nonlinearity. Therefore, very few exact solutions of unsteady problems have been obtained, and mainly to similarity problems [2-4].

There is presently an interest in high-speed gas flow, associated with the development of investigations of two-phase reacting systems. In two-phase detonation or fast convective combustion [5, 6], the relative speed of the gas and the particles can reach several hundred meters per second. To understand these processes and monitor numerical solutions in their modeling it is desirable to have accurate solutions of the unsteady equations. For the problem of expulsion of a gas from a porous medium the author has obtained asymptotic solutions describing the flow at sufficiently large time values.

1. Statement of the Problem. Ahead of the combustion front in a two-phase system there is a flow region with no chemical reaction (the filtration zone or the air plug). The friction between the gas and the particles in this region is overcome by the dynamic head of fresh combustion products.

We turn now to the following statement of the problem. A "liquid" piston, permeable for particles and impermeable for the gas, is moving according to a given law in a porous medium. We require to find the motion of the gas ahead of the piston.

We assume that there is negligible motion of the particles because of the strength of the solid or the high density. The Darcy law does not hold for high speed motion. In the free charge of particles of diameter 1 mm and flow velocity 100 m/sec the Reynolds number based on diameter is on the order of 10^4. Therefore, the main contribution to the interphase interaction does not come from the viscosity, but from the inertia of fine-scale gas flows. The real resistance law is quadratic, and we shall write it in the form $f = A\Phi u^2/d$, where u is the gas velocity; Φ is the porosity; d is the particle diameter; and A is a coefficient on the order of 1, depending on the porosity and the structure of the void space.

Since the particles are at rest, the porosity is constant. The basic equations for the gas have the form

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